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"Artifact evolution" of the axiomatic method from a "primordial soup of pictures" (with implications for "visual" language design)

Although 19th century mathematicians have largely rejected picture proof systems (Mumma, 2010), the diagrammatic reasoning community has argued for 25 years that pictures are a "valid form of reasoning" that should gain legitimacy in mathematics and computer programming language design because they afford advantages such as reducing "inferential load" (Barwise & Etchemendy, 1991) and offering "free rides" (Shimojima, 1996; Shimojima & Katagiri, 2008). Nonetheless, picture proof systems have not gained mainstream success in either field. This suggests that some property of pictures may not afford (may impede) some aspect of communication required for effective proofs.

To explore the possibility that pictures may *not* afford certain types of reasoning, I will discuss the "artifact evolution" (cf. Simon, 1993; Kirsh, 2010) of the axiomatic method from a "primordial soup" of pictures to its current, typically sentential written form. By reviewing how the axiomatic method of Euclid's *Elements* emerged from ancient land surveying practices that were more pictorial, I will argue that iconic properties of pictorial representations were suitable for conveying concrete structures (such as landforms during surveying) because of their ability to recruit lower level perceptual processing capabilities (Mandler, 2006) developed to perceive-act in a concrete physical world composed of occluded surfaces and edges, and therefore pictorial properties most effectively afford communicating concrete structures (Coppin, 2014, 2015, in press).

Although pictures can be found in the most ancient cave paintings, writing systems emerged later than pictures, often from pictographs. The axiomatic method emerged within sentential writing systems even later, reaching its current form at the time of Euclid.. In the presentation, I will present a perceptual-cognitive semiotic model that describes how symbolic properties of graphic representations convey abstract concepts with more specificity relative to pictorial properties (Coppin, 2014, 2015, in press). Then I will recruit this model to argue that pictures were too conceptually ambiguous to convey increasingly abstract/conceptual mathematical concepts that emerged when mathematics was formalized during the 19th century. As pressure for more conceptual certainty/specificity in representation systems increased, the conceptual specificity of symbolic sentential representations caused sentential writing systems to emerge as a "host" for the axiomatic method.

I will conclude by comparing the above account to Mumma's (2010) defense of Euclid's picture proofs, and demonstrate that Mumma's "co-exact" properties are akin to symbolicity (Coppin, 2014), whereas his "exact" properties are akin to iconicity. This final comparison will (i) demonstrate the accuracy of Mumma's argument, (ii) convert his terminology into cognitive semiotic terms, and (iii) use his (valid) argument to demonstrate why pictures have been unsuccessful in mathematics – or programming languages – throughout human history.

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